The peculiar velocity field in a quintessence model

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Abstract

We investigate the evolution of matter density perturbations and some properties of the peculiar velocity field for a special class of exponential potentials in a scalar field model for quintessence, for which a general exact solution is known. The data from the 2-degree Field Galaxy Redshift Survey (2dFGRS) suggest a value of the today pressureless matter density $\Omega_{\mathrm{M0}} = 0.18 \pm 0.05$.

KEYWORDS: cosmology: theory - cosmology: quintessence - large-scale structure of Universe

I. INTRODUCTION

In the last few years, a new class of cosmological models has been proposed. The more usual cosmological constant Λ-term is replaced with a dynamical, time dependent component, the quintessence or dark energy, added together with baryons, cold dark matter (CDM), photons and neutrinos. The equation of state of the dark energy is given by $w_Q \equiv \rho_Q/p_Q$, ρ_Q and p_Q being, respectively, the pressure and energy density; since $-1 \leq w_Q < 0$, it still implies a negative contribution to the total pressure of the cosmic fluid. When $w_Q = -1$, we recover a constant Λ-term. One of the more interesting physical

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realizations of the quintessence is a cosmic scalar field minimally coupled with the usual matter action [10,1]. Such a field induces the repulsive gravitational force dynamically, allowing to explain the accelerated expansion of our Universe, as discovered with observations of SuperNovae (SNe) of type Ia [11,12].

Together with an accelerated expansion, another experimental fact is the strong evidence of a spatially flat Universe [2]: quintessence could be responsible for the missing energy in a flat Universe with a subcritical matter density.

Since quintessence drives the cosmological expansion at late times, it also influences the growth of structures arisen from gravitational instability. Quintessence clusters gravitationally at very large scales ($\gtrsim 100$ Mpc), leaving an imprint on the microwave backgroud anisotropy [1]; at small scales, fluctuations in the dark energy are damped and do not enter in the evolution equation for the perturbations in the pressureless matter [5]. At the scale we are considering in the following, quintessence behaves as a smooth component: it does not partecipate directly in cluster formation, but it only alters the background cosmic evolution.

In this work, we study the evolution of density perturbations for a general exact solution for quintessence model, found by one of us together with P. Scudellaro (hereafter the RS model) [13]. In Section 2, we present the basic equations of the quintessence models examined in the paper. In Section 3, the linear perturbations equation is solved for the RS potential and the properties of the peculiar velocity field are discussed; in Section 4, the 2dFGRS data are considered. Section 5 is devoted to some final considerations.

II. MODEL DESCRIPTION

We investigate spatially flat, homogeneous and isotropic cosmologies filled with two non-interacting components: pressureless matter (dust) and a scalar field φ, minimally coupled with gravity. We consider the RS potential,

$$V(\varphi) \propto \exp\{-\sigma\varphi\}, \tag{1}$$

where $\sigma^2 \equiv 12\pi G/c^2$. We refer to Rubano & Scudellaro [13] and Pavlov et al. [7] for details and a discussion of the general exact solution of the field equations and we present here only what is needed for the present work.[1] It is

$$H = \frac{2(1 + 2\tau^2)}{3t_s\tau(1 + \tau^2)}, \tag{2}$$

and

$$\Omega_{\mathrm{M}} = \frac{1 + \tau^2}{(1 + 2\tau^2)^2}, \tag{3}$$

where $\tau \equiv t/t_s$ is a dimensionless time; t_s is a time scale of the order of the age of the Universe; H is the time dependent Hubble parameter; Ω_{M} is the pressureless matter density (ρ_M) in units of the critical density $\rho_{\mathrm{cr}} = 3H^2/8\pi G$.

The relation between the dimensionless time τ and the redshift z is given by

$$(1 + z)^3 = \frac{\tau_0(1 + \tau_0^2)}{\tau(1 + \tau^2)}, \tag{4}$$

where τ_0 is the present value of τ. This very simple cosmological model has two free parameters, t_s and τ_0, or, equivalently, H_0, the today value of the Hubble parameter, and Ω_{M0}, the present value of the pressureless matter density. As can be seen from Eqs. (3,4), Ω_{M0} depends only on τ_0

We shall compare the results for the above Universe with a flat model with constant equation of state w_Q (as well known, this includes the ΛCDM cosmologies for $w_Q = -1$). In this case, the redshift dependent Hubble parameter is

$$H(z) = H_0\sqrt{\Omega_{\mathrm{M0}}(1 + z)^3 + (1 - \Omega_{\mathrm{M0}})(1 + z)^{3(1+w_Q)}}. \tag{5}$$

This class of models has three free parameters, Ω_{M0}, H_0 and w_Q.

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[1]In particular, in Rubano & Scudellaro [13], it is explained why this potential works, on the contrary of a quite diffuse opinion.

III. PERTURBATION GROWTH AND PECULIAR VELOCITY

For perturbations inside the horizon, the equation describing the evolution of the CDM component density contrast, $\delta_M \equiv \delta\rho_M/\rho_M$ (for unclustered quintessence) is [9,5]

$$\ddot{\delta}_M + 2H(t)\dot{\delta}_M - 4\pi G\rho_M\delta_M = 0, \tag{6}$$

where the dot means derivative with respect to comoving time. In Eq. (6), the relative contribution of dark energy to the energy budget enters into the expansion rate H. We shall consider Eq. (6) in the matter dominated era, when the radiation contribution is really negligible.

With a RS potential, Eq. (6) reduces to

$$\frac{d^2\delta_M}{d\tau^2} + \frac{4}{3}\frac{(1+2\tau^2)}{\tau(1+\tau^2)}\frac{d\delta_M}{d\tau} - \frac{2}{3}\frac{1}{\tau^2(1+\tau^2)}\delta_M = 0. \tag{7}$$

Equation (7) has two linearly independent solutions, the growing mode δ_+ and the decreasing mode δ_-, which can be expressed in terms of the hypergeometric function of second type $_2F_1$. We get

$$\delta_- \propto \frac{1}{\tau} \, _2F_1\left[-\frac{1}{2},\frac{1}{3},\frac{1}{6}; -\tau^2\right], \tag{8}$$

and

$$\delta_+ \propto \tau^{2/3} \, _2F_1\left[\frac{1}{3},\frac{7}{6},\frac{11}{6}; -\tau^2\right]. \tag{9}$$

By comparing Eqs. (8,9) with Eq. (4), we observe that the density contrast, as a function of the redshift, depends on only one parameter, τ_0, i.e. Ω_{M0}. For $\tau \ll 1$, Eqs. (8,9) reduce to the standard results

$$\delta_- \propto \frac{1}{\tau}, \tag{10}$$

and

$$\delta_+ \propto \tau^{2/3}. \tag{11}$$

For the evolution of the density contrast for quintessence with constant equation of state, we refer to Silveira and Waga [15].

The linear theory relates the peculiar velocity field $\mathbf{v}$ and the density contrast by [9,6]

$$\mathbf{v}(\mathbf{x}) = H_0 \frac{f}{4\pi} \int \delta_M(\mathbf{y}) \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^3} d^3\mathbf{y}, \tag{12}$$

where the growth index f is defined as

$$f \equiv \frac{d \ln \delta_M}{d \ln a}; \tag{13}$$

a is the scale factor, $a = a_0/(1+z)$, and a_0 is its present value. For a RS potential, we get

$$f = \frac{2}{3} \frac{(1 + 2\tau^2)}{\tau(1 + \tau^2)} \frac{d \ln \delta_M}{d\tau}. \tag{14}$$

The growth index is usually approximated by $f \simeq \Omega_{\mathrm{M}}{}^{\alpha}$. For ΛCDM models, $\alpha \simeq 0.55$; if $w_Q = -2/3$, $\alpha \simeq 0.56$; if $w_Q = -1/3$, $\alpha \simeq 0.57$ [15,17]. In Fig. (1), we show the logarithm of f as a function of the logarithm of Ω_{M} for the RS potential. The value $\alpha \simeq 0.57$ provides a good approximation to the model.

In order to compare the various models, we have to stress that the "right" value for Ω_{M0} depends on the model. It is the result of a best fit prodecure on experimental data, i.e. from SN Ia observations. The well known value $\Omega_{\mathrm{M0}} \simeq 0.3$ refers only to the ΛCDM model. With the same procedure applied to SNe Ia data, the RS potential gives $\Omega_{\mathrm{M0}} = 0.15^{+0.15}_{-0.03}$ [7]. Due to the large errors, the two results are still marginally compatible, so the value $\Omega_{\mathrm{M0}} \simeq 0.3$ holds for the RS model too. In the case of a constant $w_Q \neq -1$, the situation is more complicate, due to the degeneration between Ω_{M0} and w_Q [11,14]. In this preliminary study, we decided to consider only two situations: $i)$ a ΛCDM, a $w_Q \neq -1$ and a RS model with $\Omega_{\mathrm{M0}} = 0.3$; $ii)$ the same three models with $\Omega_{\mathrm{M0}} = 0.15$. As can be seen from Fig. (2), a large variation in Ω_{M0} entails a remarkable difference in the growth index also at low redshifts. Even more remarkable is the difference at large redshifts in the case of the same Ω_{M0}. This suggest that with the deep surveys planned in the future, this approach could provide a constraint independent of SNe Ia [18].

In Fig. (3), we show the relative variations of RS models from ΛCDM cosmologies. For models with the same Ω_{M0}, the relative variations are quite small and decrease with increasing pressureless matter density. For $\Omega_{M0} = 0.3$, the relative variation is $\overset{<}{\sim} 5\%$; for $\Omega_{M0} = 0.15$, it is $\overset{<}{\sim} 15\%$.

IV. A FIRST CHECK WITH OBSERVATIONS

Low redshifts have been investigated by the 2dFGRS collaboration [8]. Peculiar velocity distorts the correlation function of galaxies observed in redshift space [4,3]. In linear theory, the characteristic quadrupole distortion enables to measure the growth index from redshift galaxy catalogs. The 2dFGRS has obtained redshifts for more than $141,000$ galaxies with an effective depth of $z = 0.17$. From a precise measurement of the clustering, a value of $\beta \equiv f/b = 0.43 \pm 0.07$ has been determined [8], where b is the bias parameter, connecting the relative density fluctuations of the galaxies and of the total mass. The bias parameter has been measured by computing the bispectrum of the 2dFGRS; it is $b = 1.04 \pm 0.11$ [16]. Combining the measurements of β and b, the growth index at $z = 0.17$ can be estimated: it is $f(z = 0.17) = 0.45 \pm 0.06$ [16]. By using the RS potential, from this value of f we can obtain an estimate of the today pressureless matter density parameter. We get

$$\Omega_{M0} = 0.18 \pm 0.05. \tag{15}$$

This value is significantly compatible with the estimate from the SN Ia data.

V. CONCLUSIONS

The present state of accuracy in observations does not allow to discriminate among the illustrated alternatives.

In our opinion, the above discussion adds another argument in favour of the use of exponential potentials in quintessence cosmology. The strong dependence of the growth

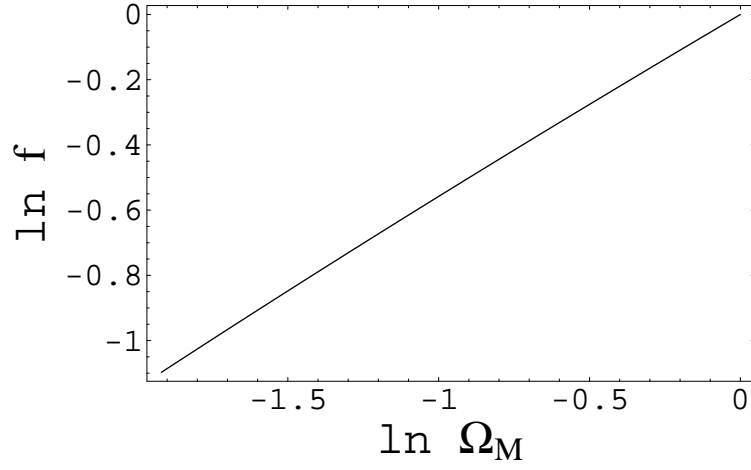index on the model, at high redshifts, suggests that more stringent constraints can be given in the future.
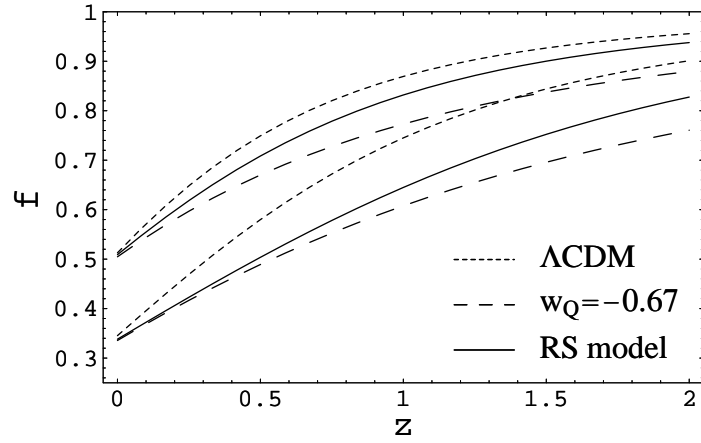
FIG. 1. $\ln f$ versus $\ln \Omega_M$ in the RS model.

FIG. 2. The growth index as a function of the redshift for two values of Ω_{M0}. The upper curves are for $\Omega_{M0} = 0.3$, the lower curves for $\Omega_{M0} = 0.15$. The dashed, long-dashed and full lines correspond, respectively, to ΛCDM cosmologies, models with $w_Q = -2/3$ and the RS model.
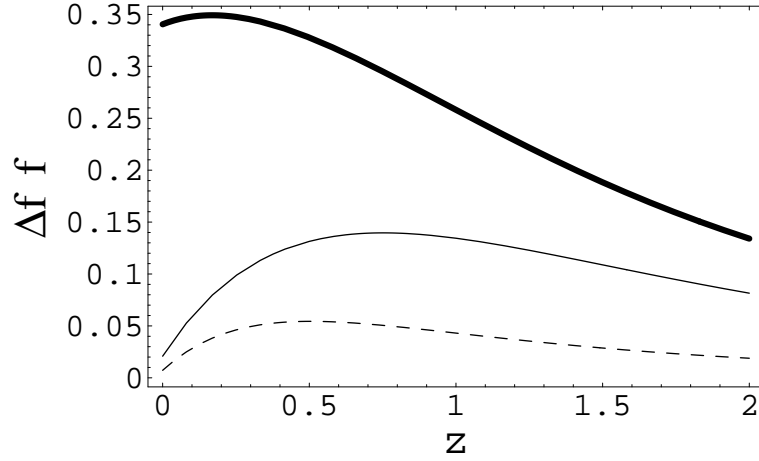
FIG. 3. Relative variations of the growth index in the RS model from the ΛCDM case as a function of the redshift. For the full thin line, $\Omega_{M0} = 0.15$; for the dashed line, $\Omega_{M0} = 0.3$. With the thick curve we indicate the variation of the RS model with $\Omega_{M0} = 0.15$ from a ΛCDM model with $\Omega_{M0} = 0.3$.

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